Location-Thinking and Value-Thinking: An Empirical Study and Framework of Two Ways Students Reason about Graphs

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Motivation & Research Question

- Visual representations of formal statements in undergraduate mathematics are commonly used in textbooks to support student understanding.
  - Ex. The Intermediate Value Theorem (IVT)

Research Question:
What are characteristics of students’ visual reasoning in the context of statements from Calculus? Specifically,
- How do students interpret outputs of the function, points on the graph, and graphs as a whole?
- How do students’ visual reasoning impact their understanding and evaluation of the Intermediate Value Theorem and similar statements?

Our Original Theoretical Framework

<table>
<thead>
<tr>
<th>Value-Thinking</th>
<th>Location-Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visual Reasoning</td>
<td>Evidence</td>
</tr>
<tr>
<td>Output of Function</td>
<td>The resulting value from inputting a value in the function</td>
</tr>
<tr>
<td></td>
<td>Labels output values on output axis</td>
</tr>
<tr>
<td>Point on Graph</td>
<td>The coordinated values of the input and output represented together</td>
</tr>
<tr>
<td></td>
<td>Labels points as ordered pairs</td>
</tr>
<tr>
<td>Graph as a Whole</td>
<td>A collection of visualized points of the input and output</td>
</tr>
<tr>
<td></td>
<td>A collection of spatial locations in the Cartesian plane associated with input values</td>
</tr>
<tr>
<td></td>
<td>Labels output on the graph</td>
</tr>
<tr>
<td></td>
<td>Labels point as output</td>
</tr>
<tr>
<td></td>
<td>Speaks about points as the result of coordinating an input and output value</td>
</tr>
</tbody>
</table>

Data Analysis & Results

- Consistent with grounded theory, we used a process of open coding and axial coding in which our visual reasoning framework emerged (Corbin & Strauss, 2014).
- We categorized each of the nine students according to our visual reasoning framework.

Methodology

- Clinical Interviews (Clement, 2000) with nine undergraduate students, three from Calculus I, Introduction to Proof, and Advanced Calculus
- Participants evaluated the following statements alone, then using various graphs below:
  1. The Intermediate Value Theorem (IVT)
  2. Extreme Value Theorem (EVT)
  3. Mean Value Theorem (MVT)

Table 1. Statements given to participants

Characteristics of Location-Thinking

- Clinical Interviews (Clement, 2000) with nine undergraduate students, three from Calculus I, Introduction to Proof, and Advanced Calculus
- Participants evaluated the following statements alone, then using various graphs below:
  1. The Intermediate Value Theorem (IVT)
  2. Extreme Value Theorem (EVT)
  3. Mean Value Theorem (MVT)

In Figure 7 (right):
- Zack, a location-thinker, considered N to be locations along the graph
- Labeled points as outputs
- Incorrectly claimed f(a) = f(b) on this constant function

In Figure 8 (left):
- Nate, another location-thinker also labeled points as outputs
- Claimed M’s label on the graph were between f(a) and f(b)
- Claimed point marked on far right as an output not between f(a) and f(b)

Significance of Findings

- Five students were classified as value-thinkers and four students as location-thinkers
- Three value-thinkers and no location-thinkers evaluated all four statements correctly
- Value-Thinking and Location-Thinking highlight distinctions in visual reasoning in students’ understandings of:
  - Claimed M’s label on the graph were between f(a) and f(b)
  - Outputs of functions
  - Points on graphs
  - Graphs as a whole
- Students may not interpret graphs as intended
- Different interpretations of graphs can have significant consequences on students’ understanding of the given concept

Implications for Instruction

- When teaching the Intermediate Value Theorem, instructors might incorporate graphs like Graph 1 to differentiate in students’ visual reasoning
- In general, when teaching how to interpret graphs in the Cartesian coordinate system, instructors might seek opportunities to distinguish between values represented at points and locations of points in space in instruction
  - For instance, when referring to and labeling outputs of a function on its graph in the Cartesian plane, refer to the value as represented on the output axis.
- Our research is situated in the context of IVT, in which value-thinking was the intended visual reasoning for students. The role of value-thinking and location-thinking, though, may be different in different contexts, such as geometry or graphs in different coordinate systems.

References